Counting Methods

1. Fundamental Counting Principle – Tree Diagram

Definition Given 2 activities A1 and A2 that can be performed in N1 and N2 different ways each, the total number of ways A1 followed by A2 can be performed is N1 x N2.

Examples

2 skirts and 3 blouses: 2 x 3 = 6
1 book from list of 5 and 1 book from list of 7: 5 x 7 = 35
Manager from 4 people and Asst Manager from 3 people: 4 x 3 = 12
2 highways from Tucson to Phoenix, 3 highways from Phoenix to Flagstaff 2x3=6

2. Permutations

Definition

1. An arrangement of items from a single set
2. Repetitions are not allowed
3. The order is significant

Examples

5 people sitting in 5 different places around a table: 5 x 4 x 3 x 2 x 1
Prizes for 1st place, 2nd place 3rd place chosen from 10 people: 10 x 9 x 8
Select Officers P, VP, Sec, Treas from 10 people: 10x9x8x7
10 students in contests, 1st, 2nd, & 3rd prizes 10 x 9 x 8
Penny, nickel, dime, and quarter to 3 children 4 x 3 x 2

Notation "8 things 3 at a time" ${}_8P_3 = 8 \times 7 \times 6$
" n things r at a time" nPr

nPr = n (n-1)(n-2) …r times

Also $nPr = \frac{n!}{(n-r)!}$ where 0! = 1

nPr can also written as P(n,r)

If there is more than one of some item

e.g. zoonooz has 2 z's and 4 o's $\frac{7!}{2!4!}$

The number of permutations in which i things are alike and another j things are alike is $\frac{n!}{i!\,j!}$ where n is the total number of things

3. Combinations

Definition

1. A selection of items from a single set
2. Repetitions are not allowed
3. The order does not matter

Examples

Select a set of 3 out of 7 books $\frac{7x6x5}{3x2x1}$

Select a committee of 4 from 10 people $\frac{10x9x8x7}{4x3x2x1}$

Notation

$$\text{nCr} = \frac{{}_nP_r}{r!}$$

Also $$\text{nCr} = \frac{n!}{r!(n-r)!}$$

nCr can also be written as C(n,r)

The number of combinations is always less than the number of permutations of a set of objects

4. More Than One Method Used Together

Examples:

1. Cafeteria offers 4 meats, 6 vegetables, 5 desserts. How many ways can a meal be served with 2 meats, 3 vegetables, and 2 desserts?

C(4,2) x C(6,3) x C(5,2) = $\frac{4x3}{2x1} x \frac{6x5x4}{3x2x1} x \frac{5x4}{2x1} = 1200$

2. Club as 14 males and 16 females. A committee of 3 men and 3 women is formed.

Male = C(14,3) = 364 = $\frac{14!}{3!(11!)}$

Female = C(16,3) = 560= $\frac{16!}{3!(13!)}$

They can be chosen 364 x 560 different ways.